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Inductive Reactance of a Rigid Wing at Harmonic Linear Oscillations and Angle of Attack

E. V. Romanenko[a], S. G. Pushkov[b], and V. N. Lopatin[a]

[a]*Severtsov Institute of Ecology and Evolution, Russian Academy of Sciences, Leninskii pr. 33, Moscow, 119071 Russia*
[b]*Gromov Flight Research Institute, Zhukovskii, Moscow Oblast, Russia*
e-mail: evromanenko33@mail.ru

Abstract—The inductive reactance of a flat and rigid wing performing harmonic oscillations with a sufficiently large amplitude at an arbitrary position of the axis of rotation was estimated. In the plane problem, analytical expressions for the components of inductive reactance through the coefficients of hydrodynamic derivatives for harmonic variations in the angle of attack were obtained.

Keywords: inductive reactance, propulsive coefficient, power factor, rigid wing, modeling

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Earlier (Romanenko and Pushkov, 1998, 2008; Pushkov and Romanenko, 2000; Pushkov et al., 2006; Romanenko et al., 2005, 2007; Romanenko, 2002), the formulas for estimating the hydrodynamic forces developed by a rigid wing oscillating in an inviscid fluid with arbitrary amplitudes of linear and angular oscillations and an arbitrary position of the axis of rotation were derived. In these formulas, the component of hydrodynamic forces that was determined by the inductive reactance of the wing was calculated by the top estimate (i.e., to the maximum) as follows:

$$X_i \le \frac{\rho\pi S v_n^2}{4}, \tag{1}$$

where S is the wing area and v_n is the normal velocity of the wing. The inductive reactance coefficient is calculated by the formula

$$C_{X_i} = \frac{\pi}{2U_0^2} v_n^2. \tag{2}$$

It is shown that expressions (1) and (2) are sufficient approximations in the calculations of the propulsive characteristics in cases of moderate aspect ratios of the wing $2 \le \lambda \le 5$ or when the percentage of inductive reactance in the overall balance of hydrodynamic forces is small. However, the problems associated with the error of the estimates used depending on the shape of the wing and kinematics remain to be solved. Previously, we derived the formulas for the inductive reactance of the wing at harmonic pitch oscillations (Pushkov et al., 2009).

In this paper, we derived the formulas for the case of harmonic oscillations of the angle of attack of the wing.

In this case, if the motion of the wing can be represented as the main motion with velocity U_0 and superposed additional motion with small a low velocity and small displacements, the general expressions for the projection of the hydrodynamic forces can be expressed as follows (Nekrasov, 1947; Romanenko, 2001; Sedov, 1966):

$$\begin{aligned} Y &= -\lambda_{22}\frac{dv_n}{dt} - \rho U_0 \Gamma, \\ X &= \lambda_{22} v_n \omega_z + \rho v_n \Gamma - \rho\pi b u_*(v_n - u_*). \end{aligned} \tag{3}$$

Here, $\Gamma = \pi b\left(v_n - \frac{b\omega_z}{4} - u_*\right)$ is the associated circulation (Pushkov and Romanenko, 2000), u_* is the effective induced velocity determined by the presence of the vortex wake of the wing, b is the wing chord, v_n is the normal velocity of the wing, ρ is the density of the medium, λ_{22} is the associated mass of the wing, and ω_z is the angular velocity of the wing.

We are interested in the third term for the projection of the hydrodynamic force X in Eq. (3) as follows:

$$X_i = \rho\pi b u_*(v_n - u_*), \tag{4}$$

which is referred to as the inductive reactance.

It should be noted that X_i is in fact only a component of the inductive reactance if it is determined by the expression

$$X_i^* = \rho\pi b u_*\left(v_n - \frac{\omega_z b}{4} - u_*\right) = \rho u_* \Gamma.$$

When determining the considered component of hydrodynamic forces X_i (4) (below, X_i is the inductive reactance), the unknown quantity is the velocity u_*. In the case of a steady or quasi-steady motion of a wing with a finite span, the vortex wake of the wing is determined mainly by the finiteness of the wing span. Velocity u_*, induced by the vortex wake, is less than v_n

in the absolute value. In this case, for aspect ratios $2 \le \lambda \le 5$, the estimation of the inductive reactance from the top gives very good results.

In the case of an infinite wing extension (the plane problem considered), the vortex wake is generated by the changes in the circulation in the presence of transverse and angular oscillations of the wing. In this case, u_* can be either greater or smaller than v_n, and X_i can be either negative or positive, respectively (recall that X_i is only a component of the inductive reactance). Velocity u_* in Eq. (4) can be determined from the ratio for the heaving force (Pushkov and Romanenko, 2000) as follows:

$$\begin{aligned} Y &= -\lambda_2 \dot{v}_n - \rho U \Gamma \\ &= -\lambda_{22} \dot{v}_n - \rho U \pi b \left(v_n - \frac{\omega_z b}{4} - u_* \right), \end{aligned} \tag{5}$$

and the expression for the heaving force through the coefficients of hydrodynamic derivatives (Belotserkovskii, 1958) is

$$Y = \frac{\rho U^2 b}{2} \left(-C_y^{\alpha} \frac{v_n}{U} - C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2} + C_y^{\omega_z} \frac{\omega_z b}{U} + C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \right). \tag{6}$$

Here, U is the instantaneous velocity of the incident flow and α is the angle of attack. The dot over the symbol denotes the time derivative.

Equating the right sides of Eqs. (5) and (6) yields

$$\begin{aligned} &\frac{\rho U^2 b}{2} \left(-C_y^{\alpha} \frac{v_n}{U} - C_y^{\dot{\alpha}} \frac{\dot{v}_n b}{U^2} + C_y^{\omega_z} \frac{\omega_z b}{U} + C_y^{\dot{\omega}_z} \frac{\dot{\omega}_z b^2}{U^2} \right) \\ &= -\lambda_{22} \dot{v}_n - \rho U \pi b \left(v_n - \frac{\omega_z b}{4} - u_* \right). \end{aligned} \tag{7}$$

Equation (7) yields the following solution to u_* (Pushkov et al., 2009):

$$\begin{aligned} u_* = v_n - \frac{v_n}{2\pi} C_y^{\alpha} + \frac{\omega_z b}{2\pi} C_y^{\omega_z} - \frac{\omega_z b}{4} + \frac{\lambda_{22} \dot{v}_n}{\rho \pi b U} \\ - \frac{\dot{v}_n b}{2\pi U} C_y^{\dot{\alpha}} + \frac{\dot{\omega}_z b^2}{2\pi U} C_y^{\dot{\omega}_z}. \end{aligned}$$

Here, all of the parameters are taken at the center of the wing. There are known solutions for the coefficients of hydrodynamic derivatives (Belotserkovskii, 1958).

Let us apply these ratios to determine the relevant components of the propulsive force and power (the members including the inductive reactance) in the case of harmonic changes in the angle of attack of the wing and its linear oscillations.

The expression for the propulsive force of the wing was derived earlier (Romanenko and Pushkov, 2008):

$$\begin{aligned} \overline{F_{xc}} = \frac{\rho S}{2} \Bigg\{ C_{yc}^{\alpha} \overline{v_{nc} V_{yc}} + b \left(C_{yc}^{\dot{\alpha}} - \frac{2m^*}{\rho S b} \right) \overline{\dot{v}_{nc} \sin\theta_c} - C_{yc}^{\dot{\omega}_z} b^2 \\ \times \overline{\dot{\omega}_z \sin\theta_c} - b C_{yc}^{\omega_z} b^2 \overline{\omega_z V_{yc}} - \overline{X_{ic} \cos\vartheta} - C \overline{U_c^2 \cos\vartheta} \Bigg\}. \end{aligned} \tag{8}$$

Here, $m^* = \lambda_{22}$.

Equation (8) can be represented in the form of propulsive coefficients as follows:

$$C_T = C_{T1} + C_{T2} + C_{T3} + C_{T4} + C_{T5} + C_{T6}. \tag{9}$$

The following expression was obtained for the power:

$$\begin{aligned} -\frac{2 F_{yc} V_{yc}}{\rho S U_0^3} - \frac{2 M_{zc} \omega_z}{\rho S U_0^3} = C_{P1} + C_{P2} + C_{P3} + C_{P4} + C_{P5} \\ + C_{P6} + C_{P7} + C_{P8} + C_{P9} + C_{P10} + C_{P11}, \end{aligned}$$

where

$$\begin{aligned} -\overline{F_{yc} V_{yc}} &= \lambda_{22} \overline{V_{yc} \frac{d(v_{nc} \cos\vartheta)}{dt}} + \frac{\rho S}{2} \\ &\times \left[\begin{aligned} C_{yc}^{\alpha} \overline{v_{nc} V_{cx} V_{yc}} + \left(C_{yc}^{\dot{\alpha}} - \frac{2\lambda_{22}}{\rho S b} \right) b \overline{\dot{v}_{nc} V_{yc} \cos\theta_c} \\ - C_{yc}^{\omega_z} \overline{\omega_z b V_{xc} V_{yc}} - C_{yc}^{\dot{\omega}_z} \overline{\dot{\omega}_z b^2 V_{yc} \cos\theta_c} \end{aligned} \right] \\ &+ \overline{X_{ic} V_{yc} \sin\vartheta} + \overline{\frac{\rho S U_c^2 V_{yc}}{2}} C \sin\vartheta \end{aligned}$$

$$\begin{aligned} -\overline{M_{zc} \omega_z} &= \frac{\rho S b}{2} \\ &\times \left[m_{zc}^{\alpha} \overline{\alpha_c \omega_z U_c^2} + m_{zc}^{\dot{\alpha}} \overline{\frac{\dot{\alpha}_c b \omega_z U_c^2}{U_0}} - m_{zc}^{\omega_z} \overline{\frac{\omega_z^2 b U_c^2}{U_0}} \right. \\ &\left. - m_{zc}^{\dot{\omega}_z} \overline{\frac{\dot{\omega}_z \omega_z b^2 U_c^2}{U_0^2}} \right]. \end{aligned}$$

Hereinafter, $\bar{F}_{xc}$ is the propulsive force, F_{yc} is the vertical force, M_{zc} is the moment, λ_{22} is the associated mass of the wing, v_{nc} is normal velocity, ρ is the density of the medium, θ_c is the angle between the incident flow on the wing and the horizontal axis, C is the reactance coefficient of the wing, U_c is the instantaneous velocity of the incident flow on the wing, X_{ic} is the inductive reactance of the wing, b is the wing chord, and S is the wing area (one side). C_{yc}^{α}, $C_{yc}^{\alpha'}$, $C_{yc}^{\omega_z}$, and $C_{yc}^{\omega_z'}$ are the aerodynamic derivatives, and m_{zc}^{α}, $m_{zc}^{\dot{\alpha}}$, $m_{zc}^{\omega_z}$, and $m_{zc}^{\dot{\omega}_z}$ are the moment derivatives (Belotserkovskii, 1958). The presence of index "c" means that the values were recalculated to the center of the wing.

One component of the propulsive coefficient, which includes the inductive reactance, has the following form:

$$C_{T5} = -\frac{\overline{2X_{ic}\cos\vartheta}}{\rho S U_0^2}. \tag{10}$$

Below, ϑ is the pitch to the horizontal axis.

Hence, having expanded expression (10), we obtain the equation

$$C_{T5} = -\frac{2\pi}{U_0^2}\Bigg(D_1\overline{v_{nc}^2\cos\vartheta} + D_2\overline{v_{nc}\omega_z\cos\vartheta} + D_3\overline{\frac{v_{nc}\dot{\omega}_z}{U_c}\cos\vartheta} + D_4\overline{\frac{v_{nc}\dot{v}_{nc}}{U_c}\cos\vartheta} + D_5\overline{\frac{v_{nc}\omega_z}{U_c}\cos\vartheta} + D_6\overline{\omega_z^2\cos\vartheta} + D_7\overline{\frac{\omega_z\dot{\omega}_z}{U_c}\cos\vartheta} + D_8\overline{\frac{\dot{v}_{nc}^2}{U_c^2}\cos\vartheta} + D_9\overline{\frac{\dot{v}_{nc}^2\dot{\omega}_z}{U_c^2}\cos\vartheta} + D_{10}\overline{\frac{\dot{\omega}_z^2}{U_c^2}\cos\vartheta}\Bigg). \tag{11}$$

Factor $2\pi D_1$–$2\pi D_{10}$ are given elsewhere (Pushkov et al., 2009).

Consider the case of harmonic linear oscillations of an infinite wing and its angle of attack. In this case, $y = y_0\sin\omega t$ and $\alpha = \alpha_0\cos\omega t$; the phase shift between the linear and angular oscillations was assumed to be 90°. Variables in expression (11) have the following form:

$$v_{nc} = V_{y1}\cos\vartheta - U_0\sin\vartheta + \omega_z x = \alpha_c U_c,$$

$$U_c^2 = V_{yc}^2 + V_{xc}^2,$$

$$V_{xc} = U_0 - \omega_z x\sin\vartheta,$$

$$V_{yc} = V_{y1} + \omega_z x\cos\vartheta,$$

where $V_{y1} = \dot{y}(t)$, $\omega_z = \dot{\vartheta}(t)$, and $y(t)$ are vertical oscillations of the wing (dots over the symbol denote the time derivatives). The pitch of the wing to the horizontal axis (ϑ) is set by the expression

$$\vartheta = \theta_1 - \alpha_1.$$

The pitch of the wing does not have index c, since it is the same at all points of the wing, including point x_1 (see the figure in (Pushkov and Romanenko, 2000)). Therefore, it is determined by the kinematic parameters of this particular point, i.e., by the instantaneous angle of the incoming flow θ_1 and the angle of attack α_1 at x_1. Values $\sin\vartheta$ and $\cos\vartheta$ in formulas (3) and (4), with allowance for (5) and the smallness of the angle of attack, can be written in the form

$$\sin\vartheta \approx \sin\theta_1 - \alpha_1\cos\theta_1,$$

$$\cos\vartheta \approx \cos\theta_1 + \alpha_1\sin\theta_1.$$

Here, $\theta_1 = \vartheta + \alpha_1 = \arctan\frac{V_{y1}}{U_0}$, $\cos\theta_1 = \frac{U_0}{U_1}$, $\sin\theta_1 = \frac{y_0\omega\cos\omega t}{U_1}$, and $U_1 = \sqrt{U_0^2 + (y_0\omega)^2\cos^2\omega t}$.

Expression (11) can be written as

$$C_{T5} = C_{T5-1} + C_{T5-2} + C_{T5-3} + C_{T5-4} + C_{T5-5} + C_{T5-6} \\ C_{T5-7} + C_{T5-8} + C_{T5-9} + C_{T5-10}. \tag{12}$$

In this case, members of the right side of Eq. (12) have the form

$$C_{T5-1} = -2\pi D_1\frac{\overline{v_{nc}^2\cos\vartheta}}{U_0^2} = -2\pi D_1\frac{2\sqrt{2}(Sh_0)^2\lambda_P^2X^2}{(2\lambda_P^2+1)\sqrt{2\lambda_P^2+1}}J_{5-1},$$

$$J_{5-1} = \Bigg[\frac{\lambda_P}{(2\lambda_P^2+1)}J_{5-1-1} - \alpha_0 J_{5-1-2} + \frac{\alpha_0}{2(2\lambda_P^2+1)}J_{5-1-3} + \frac{\alpha_0^2(2\lambda_P^2+1)^2}{8\lambda_P^3(Sh_0)^2X^2}J_{5-1-4} + \frac{\alpha_0^2(2\lambda_P^2+1)}{4\lambda_P}J_{5-1-5} + \frac{\alpha_0^3(2\lambda_P^2+1)^2}{16(Sh_0)^2\lambda_P^4X^2}J_{5-1-6} - \frac{\alpha_0^2}{2\lambda_P}J_{5-1-7} + \frac{\alpha_0^3(2\lambda_P^2+1)}{8\lambda_P^2}J_{5-1-8}\Bigg],$$

$$J_{5-1-1} = \Bigg[1 + \frac{1.25}{(2\lambda_P^2+1)} + \frac{2.188}{(2\lambda_P^2+1)^2} + \frac{2.461}{(2\lambda_P^2+1)^3} + \frac{3.384}{(2\lambda_P^2+1)^4} + \frac{2.964}{(2\lambda_P^2+1)^5} + \frac{2.795}{(2\lambda_P^2+1)^6} + \frac{2.094}{(2\lambda_P^2+1)^7} + \frac{1.57}{(2\lambda_P^2+1)^8}\Bigg],$$

$$J_{5-1-2} = \Bigg[1 + \frac{0.75}{(2\lambda_P^2+1)} + \frac{0.938}{(2\lambda_P^2+1)^2} + \frac{0.82}{(2\lambda_P^2+1)^3} + \frac{0.923}{(2\lambda_P^2+1)^4} + \frac{0.457}{(2\lambda_P^2+1)^5} + \frac{0.3}{(2\lambda_P^2+1)^6} + \frac{0.16}{(2\lambda_P^2+1)^7} + \frac{0.075}{(2\lambda_P^2+1)^8}\Bigg],$$

$$J_{5-1-3} = \left[0.5 + \frac{0.547}{(2\lambda_P^2+1)^2} + \frac{0.564}{(2\lambda_P^2+1)^4} + \frac{0.349}{(2\lambda_P^2+1)^6} + \frac{0.157}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-1-4} = \left[1 + \frac{0.25}{(2\lambda_P^2+1)} - \frac{0.0625}{(2\lambda_P^2+1)^2} + \frac{0.0234}{(2\lambda_P^2+1)^3} - \frac{0.0146}{(2\lambda_P^2+1)^4} + \frac{0.0085}{(2\lambda_P^2+1)^5}\right],$$

$$J_{5-1-5} = \left[1 + \frac{0.25}{(2\lambda_P^2+1)} + \frac{0.1875}{(2\lambda_P^2+1)^2} + \frac{0.1172}{(2\lambda_P^2+1)^3} + \frac{0.1025}{(2\lambda_P^2+1)^4}\right],$$

$$J_{5-1-6} = \left[1.5 + \frac{0.5}{(2\lambda_P^2+1)} - \frac{0.1094}{(2\lambda_P^2+1)^2} + \frac{0.0469}{(2\lambda_P^2+1)^3} - \frac{0.0286}{(2\lambda_P^2+1)^4} + \frac{0.0171}{(2\lambda_P^2+1)^5}\right],$$

$$J_{5-1-7} = \left[0.5 + \frac{0.2344}{(2\lambda_P^2+1)^2} + \frac{0.1538}{(2\lambda_P^2+1)^4} + \frac{0.0375}{(2\lambda_P^2+1)^6} + \frac{0.0075}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-1-8} = 0.5 + \frac{0.0469}{(2\lambda_P^2+1)^2} + \frac{0.0171}{(2\lambda_P^2+1)^4}.$$

$$C_{T5-2} = -2\pi D_2\left(\frac{\overline{\nu_{nc}\omega_z\cos\vartheta}}{U_0^2}\right) = -2\pi D_2\frac{2\sqrt{2}(Sh_0)^2\lambda_P^3X}{(2\lambda_P^2+1)^2\sqrt{(2\lambda_P^2+1)}}J_{5-2},$$

$$J_{5-2} = \left(J_{5-1-1} - \frac{\alpha_0(2\lambda_P^2+1)}{2\lambda_P}J_{5-1-2} + \frac{\alpha_0^2(2\lambda_P^2+1)^2}{4\lambda_P^2}J_{5-1-5} + \frac{\alpha_0}{2\lambda_P}J_{5-1-3} - \frac{\alpha_0^2(2\lambda_P^2+1)}{4\lambda_P^2}J_{5-1-7} + \frac{\alpha_0^3(2\lambda_P^2+1)^2}{8\lambda_P^2}J_{5-1-8}\right).$$

$$C_{T5-3} = -2\pi D_3\frac{\overline{\nu_{nc}\dot{\omega}_z\cos\vartheta}}{U_0^2U_c^2} = -2\pi D_3\left[-\frac{\sqrt{2}(Sh_0)^2\alpha_0\lambda_P^2}{(2\lambda_P^2+1)\sqrt{(2\lambda_P^2+1)}}\right]J_{5-3},$$

$$J_{5-3} = \left[J_{5-3-1} + \frac{2}{(2\lambda_P^2+1)}J_{5-1-3} - \frac{\alpha_0(2\lambda_P^2+1)}{2\lambda_P}J_{5-3-3} + \frac{\alpha_0}{2\lambda_P}J_{5-3-4} + \frac{\alpha_0}{\lambda_P(2\lambda_P^2+1)}J_{5-3-5} - \frac{\alpha_0^2(2\lambda_P^2+1)}{4\lambda_P^2}J_{5-3-6}\right],$$

$$J_{5-3-1} = \left[1 - \frac{0.75}{(2\lambda_P^2+1)} + \frac{0.938}{(2\lambda_P^2+1)^2} - \frac{0.82}{(2\lambda_P^2+1)^3} + \frac{0.923}{(2\lambda_P^2+1)^4} - \frac{0.457}{(2\lambda_P^2+1)^5} + \frac{0.3}{(2\lambda_P^2+1)^6} - \frac{0.16}{(2\lambda_P^2+1)^7} + \frac{0.075}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-3-3} = \left[1 - \frac{0.25}{(2\lambda_P^2+1)} + \frac{0.1875}{(2\lambda_P^2+1)^2} - \frac{0.1172}{(2\lambda_P^2+1)^3} + \frac{0.1025}{(2\lambda_P^2+1)^4}\right],$$

$$J_{5-3-4} = \left[1.5 - \frac{1.5}{(2\lambda_P^2+1)} + \frac{1.64}{(2\lambda_P^2+1)^2} - \frac{1.64}{(2\lambda_P^2+1)^3} + \frac{1.6919}{(2\lambda_P^2+1)^4} - \frac{0.9131}{(2\lambda_P^2+1)^5} + \frac{0.5628}{(2\lambda_P^2+1)^6} - \frac{0.32}{(2\lambda_P^2+1)^7} + \frac{0.1423}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-3-5} = \left[0.5 - \frac{0.3125}{(2\lambda_P^2+1)} + \frac{0.5469}{(2\lambda_P^2+1)^2} - \frac{0.41}{(2\lambda_P^2+1)^3} + \frac{0.564}{(2\lambda_P^2+1)^4} - \frac{0.37}{(2\lambda_P^2+1)^5} + \frac{0.3494}{(2\lambda_P^2+1)^6} - \frac{0.209}{(2\lambda_P^2+1)^7} + \frac{0.157}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-3-6} = 1.5 - \frac{0.5}{(2\lambda_P^2+1)} + \frac{0.3281}{(2\lambda_P^2+1)^2} - \frac{0.2344}{(2\lambda_P^2+1)^3} + \frac{0.1879}{(2\lambda_P^2+1)^4}.$$

$$C_{T5-4} = 0.$$

$$C_{T5-5} = -2\pi D_5\frac{\overline{\dot{\nu}_{nc}\omega_z\cos\vartheta}}{U_0^2U_c} = -2\pi D_5\left[\frac{\sqrt{2}(Sh_0)^2\alpha_0\lambda_P^2}{(2\lambda_P^2+1)\sqrt{(2\lambda_P^2+1)}}\right]J_{5-5},$$

$$J_{5-5} = \left[J_{5-1-2} - \frac{\alpha_0(2\lambda_p^2+1)}{2\lambda_P}J_{5-1-5} + \frac{1}{(2\lambda_p^2+1)}J_{5-1-3} - \frac{\alpha_0^2(2\lambda_P^2+1)}{4\lambda_P^2}J_{5-1-8} + \frac{\alpha_0}{2\lambda_P(2\lambda_P^2+1)}J_{5-5-7} - \frac{\alpha_0^2}{4\lambda_P^2}J_{5-5-8}\right],$$

$$J_{5-5-7} = \left[0.5 - \frac{0.3125}{(2\lambda_P^2+1)} + \frac{0.5469}{(2\lambda_P^2+1)^2} - \frac{0.41}{(2\lambda_P^2+1)^3} + \frac{0.564}{(2\lambda_P^2+1)^4} - \frac{0.37}{(2\lambda_P^2+1)^5} + \frac{0.3494}{(2\lambda_P^2+1)^6} - \frac{0.2094}{(2\lambda_P^2+1)^7} + \frac{0.157}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-5-8} = \left[0.5 - \frac{0.1875}{(2\lambda_P^2+1)} + \frac{0.2344}{(2\lambda_P^2+1)^2} - \frac{0.1367}{(2\lambda_P^2+1)^3} + \frac{0.1538}{(2\lambda_P^2+1)^4} - \frac{0.0571}{(2\lambda_P^2+1)^5} + \frac{0.0375}{(2\lambda_P^2+1)^6}\right].$$

$$C_{T5-6} = -2\pi D_6(\overline{\omega_z^2\cos\vartheta}) = -2\pi D_6\left[\frac{2\sqrt{2}(Sh_0)^2\lambda_P^3}{(2\lambda_P^2+1)^2\sqrt{(2\lambda_P^2+1)}}\right]J_{5-6},$$

$$J_{5-6} = \left[J_{5-6-1} - \frac{\alpha_0(2\lambda_p^2+1)}{\lambda_P}J_{5-6-2} + \frac{\alpha_0^2(2\lambda_p^2+1)^2}{4\lambda_P^2}J_{5-6-3} + \frac{\alpha_0}{2\lambda_P}J_{5-6-4} - \frac{\alpha_0^2(2\lambda_P^2+1)}{2\lambda_P^2}J_{5-6-5} + \frac{\alpha_0^3(2\lambda_P^2+1)^2}{8\lambda_P^3}J_{5-6-6}\right].$$

$$J_{5-6-1} = J_{5-1-1},$$
$$J_{5-6-2} = J_{5-1-2},$$
$$J_{5-6-3} = J_{5-1-5},$$
$$J_{5-6-4} = J_{5-1-3},$$
$$J_{5-6-5} = J_{5-1-7},$$
$$J_{5-6-6} = J_{5-1-8},$$

$$C_{T5-7} = 0.$$

$$C_{T5-8} = -2\pi D_8\left[\frac{\overline{\dot{v}_{nc}^2\cos\vartheta}}{U_0^2U_c^2}\right] = -2\pi D_8\left[\frac{4\sqrt{2}(Sh_0)^4\lambda_P^5X^2}{(2\lambda_P^2+1)^3\sqrt{(2\lambda_P^2+1)}}\right]J_{5-8},$$

$$J_{5-8} = \left[J_{5-8-1} + \frac{4}{(2\lambda_P^2+1)}J_{5-8-2} + \frac{4}{(2\lambda_P^2+1)^2}J_{5-8-3} - \frac{\alpha_0(2\lambda_P^2+1)}{\lambda_P}J_{5-8-4} - \frac{2\alpha_0}{\lambda_P}J_{5-8-5} + \frac{\alpha_0}{2\lambda_P}J_{5-8-6} + \frac{2\alpha_0}{\lambda_P(2\lambda_P^2+1)}J_{5-8-7} + \frac{2\alpha_0}{\lambda_P(2\lambda_P^2+1)^2}J_{5-8-8} + \frac{\alpha_0^3(2\lambda_P^2+1)}{16(Sh_0)^2\lambda_P^5X^2}J_{5-8-9} - \frac{\alpha_0^2}{\lambda_P^2}J_{5-8-10} - \frac{\alpha_0^2(2\lambda_P^2+1)}{2\lambda_P^2}J_{5-8-11} + \frac{\alpha_0^2(2\lambda_P^2+1)}{8(Sh_0)^2\lambda_P^4X^2}J_{5-8-12} + \frac{\alpha_0^2(2\lambda_P^2+1)^3}{8(Sh_0)^2\lambda_P^4X^2}J_{5-8-13} + \frac{\alpha_0^2(2\lambda_P^2+1)^2}{4(Sh_0)\lambda_P^4X^2}J_{5-8-14} + \frac{\alpha_0^2(2\lambda_P^2+1)^2}{4\lambda_P^2}J_{5-8-15} + \frac{\alpha_0^3(2\lambda_P^2+1)^3}{16(Sh_0)^2\lambda_P^5X^2}J_{5-8-16} + \frac{\alpha_0^3(2\lambda_P^2+1)^2}{4(Sh_0)^2\lambda_P^5X^2}J_{5-8-17} + \frac{\alpha_0^3(2\lambda_P^2+1)^2}{8\lambda_P^3}J_{5-8-18}\right],$$

$$J_{5-8-1} = \left[1 - \frac{1.75}{(2\lambda_P^2+1)} + \frac{3.9375}{(2\lambda_P^2+1)^2} - \frac{5.4143}{(2\lambda_P^2+1)^3} + \frac{8.798}{(2\lambda_P^2+1)^4} - \frac{10.139}{(2\lambda_P^2+1)^5} + \frac{12.48}{(2\lambda_P^2+1)^6} - \frac{12.25}{(2\lambda_P^2+1)^7} + \frac{12.58}{(2\lambda_P^2+1)^8} - \frac{10.295}{(2\lambda_P^2+1)^9} + \frac{8.44}{(2\lambda_P^2+1)^{10}}\right],$$

$$J_{5-8-2} = \left[0.5 + \frac{1.547}{(2\lambda_P^2+1)^2} + \frac{3.142}{(2\lambda_P^2+1)^4} + \frac{4.616}{(2\lambda_P^2+1)^6} + \frac{5.1}{(2\lambda_P^2+1)^8} + \frac{4.064}{(2\lambda_P^2+1)^{10}}\right],$$

$$J_{5-8-3} = \left[0.5 + \frac{0.688}{(2\lambda_P^2+1)} + \frac{2.234}{(2\lambda_P^2+1)^2} + \frac{2.793}{(2\lambda_P^2+1)^3} + \frac{5.935}{(2\lambda_P^2+1)^4} + \frac{6.889}{(2\lambda_P^2+1)^5} + \frac{11.403}{(2\lambda_P^2+1)^6} + \frac{12.027}{(2\lambda_P^2+1)^7} + \frac{16.68}{(2\lambda_P^2+1)^8} + \frac{15.17}{(2\lambda_P^2+1)^9} + \frac{16.64}{(2\lambda_P^2+1)^{10}}\right],$$

$$J_{5-8-4} = \left[1 - \frac{1.25}{(2\lambda_P^2+1)} + \frac{2.188}{(2\lambda_P^2+1)^2} - \frac{2.461}{(2\lambda_P^2+1)^3}\right.$$

$$+\frac{3.384}{(2\lambda_P^2+1)^4}-\frac{2.964}{(2\lambda_P^2+1)^5}+\frac{2.795}{(2\lambda_P^2+1)^6}-\frac{2.094}{(2\lambda_P^2+1)^7}+\frac{1.57}{(2\lambda_P^2+1)^8}\Bigg],$$

$$J_{5-8-5} = \Bigg[0.5+\frac{0.984}{(2\lambda_P^2+1)^2}+\frac{1.466}{(2\lambda_P^2+1)^4}+\frac{1.561}{(2\lambda_P^2+1)^6}+\frac{1.258}{(2\lambda_P^2+1)^8}+\frac{0.7}{(2\lambda_P^2+1)^{10}}\Bigg],$$

$$J_{5-8-6} = \Bigg[1.5-\frac{3.5}{(2\lambda_P^2+1)}+\frac{6.891}{(2\lambda_P^2+1)^2}-\frac{10.829}{(2\lambda_P^2+1)^3}+\frac{16.13}{(2\lambda_P^2+1)^4}-\frac{20.278}{(2\lambda_P^2+1)^5}+\frac{23.409}{(2\lambda_P^2+1)^6}-\frac{24.5}{(2\lambda_P^2+1)^7}+\frac{23.898}{(2\lambda_P^2+1)^8}-\frac{20.59}{(2\lambda_P^2+1)^9}+\frac{16.176}{(2\lambda_P^2+1)^{10}}\Bigg],$$

$$J_{5-8-7} = \Bigg[0.5-\frac{0.563}{(2\lambda_P^2+1)}+\frac{1.547}{(2\lambda_P^2+1)^2}-\frac{1.676}{(2\lambda_P^2+1)^3}+\frac{3.142}{(2\lambda_P^2+1)^4}-\frac{3.192}{(2\lambda_P^2+1)^5}+\frac{4.616}{(2\lambda_P^2+1)^6}-\frac{4.241}{(2\lambda_P^2+1)^7}+\frac{5.103}{(2\lambda_P^2+1)^8}-\frac{4.131}{(2\lambda_P^2+1)^9}+\frac{4.064}{(2\lambda_P^2+1)^{10}}\Bigg],$$

$$J_{5-8-8} = \Bigg[0.375+\frac{1.117}{(2\lambda_P^2+1)^2}+\frac{2.266}{(2\lambda_P^2+1)^4}+\frac{3.421}{(2\lambda_P^2+1)^6}+\frac{4.17}{(2\lambda_P^2+1)^8}+\frac{3.56}{(2\lambda_P^2+1)^{10}}\Bigg],$$

$$J_{5-8-9} = \Bigg[0.625-\frac{0.625}{(2\lambda_P^2+1)}+\frac{0.82}{(2\lambda_P^2+1)^2}-\frac{0.82}{(2\lambda_P^2+1)^3}+\frac{0.9165}{(2\lambda_P^2+1)^4}-\frac{0.741}{(2\lambda_P^2+1)^5}+\frac{0.592}{(2\lambda_P^2+1)^6}-\frac{0.4187}{(2\lambda_P^2+1)^7}+\frac{0.2748}{(2\lambda_P^2+1)^8}\Bigg],$$

$$J_{5-8-10} = \Bigg[0.5-\frac{0.4375}{(2\lambda_P^2+1)}+\frac{0.9844}{(2\lambda_P^2+1)^2}-\frac{0.9023}{(2\lambda_P^2+1)^3}+\frac{1.4663}{(2\lambda_P^2+1)^4}-\frac{1.2677}{(2\lambda_P^2+1)^5}+\frac{1.561}{(2\lambda_P^2+1)^6}-\frac{1.2255}{(2\lambda_P^2+1)^7}+\frac{1.1845}{(2\lambda_P^2+1)^8}\Bigg],$$

$$J_{5-8-11} = \Bigg[1.5-\frac{2.5}{(2\lambda_P^2+1)}+\frac{3.8281}{(2\lambda_P^2+1)^2}-\frac{4.9219}{(2\lambda_P^2+1)^3}+\frac{6.2036}{(2\lambda_P^2+1)^4}-\frac{5.9277}{(2\lambda_P^2+1)^5}+\frac{5.2414}{(2\lambda_P^2+1)^6}-\frac{4.187}{(2\lambda_P^2+1)^7}+\frac{2.9831}{(2\lambda_P^2+1)^8}\Bigg],$$

$$J_{5-8-12} = \Bigg[0.5-\frac{0.3125}{(2\lambda_P^2+1)}+\frac{0.5469}{(2\lambda_P^2+1)^2}-\frac{0.41}{(2\lambda_P^2+1)^3}+\frac{0.564}{(2\lambda_P^2+1)^4}-\frac{0.37}{(2\lambda_P^2+1)^5}+\frac{0.3494}{(2\lambda_P^2+1)^6}-\frac{0.2094}{(2\lambda_P^2+1)^7}+\frac{0.157}{(2\lambda_P^2+1)^8}\Bigg],$$

$$J_{5-8-13} = J_{5-1-5},$$
$$J_{5-8-14} = J_{5-1-7},$$
$$J_{5-8-15} = J_{5-3-1},$$
$$J_{5-8-16} = J_{5-1-8},$$
$$J_{5-8-17} = J_{5-1-7},$$
$$J_{5-8-18} = J_{5-3-3}.$$

$$C_{T5-9} = -2\pi D_9\left[\frac{\overline{\dot{\nu}_{nc}\dot{\omega}_z\cos\vartheta}}{U_0^2U_c^2}\right] = -2\pi D_9\left[\frac{4\sqrt{2}(Sh_0)^4\lambda_P^5X}{(2\lambda_P^2+1)^3\sqrt{(2\lambda_P^2+1)}}\right]J_{5-9},$$

$$J_{5-9} = \Bigg[J_{5-9-1}+\frac{4}{(2\lambda_P^2+1)}J_{5-9-2}-\frac{\alpha_0(2\lambda_P^2+1)}{\lambda_P}J_{5-9-3}+\frac{4}{(2\lambda_P^2+1)^2}J_{5-9-4}-\frac{\alpha_0}{2\lambda_P}J_{5-9-5}+\frac{\alpha_0}{2\lambda_P}J_{5-9-6}+\frac{16\alpha_0}{\lambda_P(2\lambda_P^2+1)}J_{5-9-7}+\frac{2\alpha_0}{\lambda_P(2\lambda_P^2+1)^2}J_{5-9-8}-\frac{\alpha_0^2(2\lambda_P^2+1)}{4\lambda_P^2}J_{5-9-9}-\frac{\alpha_0^2}{2\lambda_P^2}J_{5-9-10}+\frac{\alpha_0^2(2\lambda_P^2+1)^3}{4\lambda_P^2}J_{5-9-11}+\frac{\alpha_0^3(2\lambda_P^2+1)^2}{8\lambda_P^3}J_{5-9-12}\Bigg],$$

$$J_{5-9-1} = J_{5-8-1},$$
$$J_{5-9-2} = J_{5-8-2},$$
$$J_{5-9-3} = J_{5-8-4},$$
$$J_{5-9-4} = J_{5-8-3},$$
$$J_{5-9-5} = J_{5-8-5},$$
$$J_{5-9-6} = J_{5-8-6},$$
$$J_{5-9-7} = J_{5-8-7},$$
$$J_{5-9-8} = J_{5-8-8},$$
$$J_{5-9-9} = J_{5-8-11},$$

$$J_{5-9-10} = J_{5-8-10},$$
$$J_{5-9-11} = J_{5-3-1},$$
$$J_{5-9-12} = J_{5-3-3}.$$

$$C_{T5-10} = -2\pi D_{10}\left[\frac{\overline{\dot{\omega}_z^2\cos\vartheta}}{U_0^2U_c^2}\right]$$
$$= -2\pi D_{10}\left[\frac{4\sqrt{2}(Sh_0)^4\lambda_P^5}{(2\lambda_P^2+1)^3\sqrt{(2\lambda_P^2+1)}}\right]J_{5-10},$$

$$J_{5-10} = \left[J_{5-10-1} + \frac{4}{(2\lambda_P^2+1)}J_{5-10-2} + \frac{4}{(2\lambda_P^2+1)^2}J_{5-10-3}\right.$$
$$- \frac{\alpha_0(2\lambda_P^2+1)}{\lambda_P}J_{5-10-4} - \frac{2\alpha_0}{\lambda_P}J_{5-10-5} + \frac{\alpha_0}{2\lambda_P}J_{5-10-6}$$
$$+ \frac{2\alpha_0}{\lambda_P(2\lambda_P^2+1)}J_{5-10-7} + \frac{2\alpha_0}{\lambda_P(2\lambda_P^2+1)^2}J_{5-10-8}$$
$$- \frac{\alpha_0^2(2\lambda_P^2+1)}{2\lambda_P^2}J_{5-10-9} - \frac{\alpha_0^2}{\lambda_P^2}J_{5-10-10}$$
$$\left.+ \frac{\alpha_0^2(2\lambda_P^2+1)^2}{4\lambda_P^2}J_{5-10-11} + \frac{\alpha_0^3(2\lambda_P^2+1)^2}{8\lambda_P^3}J_{5-10-12}\right],$$

$$J_{5-10-1} = J_{5-9-1},$$
$$J_{5-10-2} = J_{5-9-2},$$

$$J_{5-10-3} = \left[0.5 + \frac{0.688}{(2\lambda_P^2+1)} + \frac{1.5469}{(2\lambda_P^2+1)^2} + \frac{2.793}{(2\lambda_P^2+1)^3}\right.$$
$$+ \frac{5.1421}{(2\lambda_P^2+1)^4} + \frac{6.882}{(2\lambda_P^2+1)^5} + \frac{11.4}{(2\lambda_P^2+1)^6}$$
$$\left.+ \frac{12.027}{(2\lambda_P^2+1)^7} + \frac{16.68}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-10-4} = J_{5-9-3},$$

$$J_{5-10-5} = \left[0.5 + \frac{0.9844}{(2\lambda_P^2+1)^2} + \frac{1.4663}{(2\lambda_P^2+1)^4}\right.$$
$$\left.+ \frac{1.561}{(2\lambda_P^2+1)^6} + \frac{1.1845}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-10-6} = J_{5-9-6},$$
$$J_{5-10-7} = J_{5-9-7},$$
$$J_{5-10-8} = J_{5-9-8},$$

$$J_{5-10-9} = \left[1.5 - \frac{2.5}{(2\lambda_P^2+1)} + \frac{3.8281}{(2\lambda_P^2+1)^2} - \frac{4.9219}{(2\lambda_P^2+1)^3}\right.$$
$$+ \frac{6.2036}{(2\lambda_P^2+1)^4} - \frac{5.9277}{(2\lambda_P^2+1)^5} + \frac{5.2414}{(2\lambda_P^2+1)^6}$$
$$\left.- \frac{4.187}{(2\lambda_P^2+1)^7} + \frac{2.9831}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-10-10} = \left[0.5 - \frac{0.4375}{(2\lambda_P^2+1)} + \frac{0.9844}{(2\lambda_P^2+1)^2} - \frac{0.9023}{(2\lambda_P^2+1)^3}\right.$$
$$+ \frac{1.4663}{(2\lambda_P^2+1)^4} - \frac{1.2677}{(2\lambda_P^2+1)^5} + \frac{1.561}{(2\lambda_P^2+1)^6}$$
$$\left.- \frac{1.2255}{(2\lambda_P^2+1)^7} + \frac{1.1845}{(2\lambda_P^2+1)^8}\right],$$

$$J_{5-10-11} = J_{5-3-1},$$
$$J_{5-10-12} = J_{5-10-9}.$$

One of the components of the power factor, which includes the inductive reactance, has the form

$$C_{P6} = \frac{2}{\rho S U_0^3}\overline{X_{ic}V_{yc}\sin\vartheta}. \qquad (13)$$

Expanding expression (13), we obtain

$$C_{P6} = \frac{2\pi}{U_0^3}\Big(D_1\overline{V_{yc}\nu_{nc}^2\sin\vartheta} + D_2\overline{V_{yc}\nu_{nc}\omega_z\sin\vartheta}$$
$$+ D_3\overline{\frac{V_{yc}\nu_{nc}\dot{\omega}_z}{U_c}\sin\vartheta} + D_4\overline{\frac{V_{nc}\nu_{nc}\dot{\nu}_{nc}}{U_c}\sin\vartheta} \qquad (14)$$
$$+ D_5\overline{\frac{V_{yc}\dot{\nu}_{nc}\omega_z}{U_c}\sin\vartheta} + D_6\overline{V_{yc}\omega_z^2\sin\vartheta} + D_7\overline{\frac{V_{yc}\omega_z\dot{\omega}_z}{U_c}\sin\vartheta}$$
$$+ D_8\overline{\frac{V_{yc}\dot{\nu}_{nc}^2}{U_c^2}\sin\vartheta} + D_9\overline{\frac{V_{yc}\dot{\nu}_{nc}\dot{\omega}_z}{U_c^2}\sin\vartheta} + D_{10}\overline{\frac{V_{yc}\dot{\omega}_z^2}{U_c^2}\sin\vartheta}\Big).$$

This expression can be written as

$$C_{P6} = C_{P6-1} + C_{P6-2} + C_{P6-3} + C_{P6-4} + C_{P6-5} + C_{P6-6}$$
$$+ C_{P6-7} + C_{P6-8} + C_{P6-9} + C_{P6-10}.$$

The coefficients in the right part of the above expression are as follows:

$$C_{P6-1} = 2\pi D_1\left[\frac{\overline{V_{yc}\nu_{nc}^2\sin\vartheta}}{U_0^3}\right]$$
$$= 2\pi D_1\left[\frac{\sqrt{2}(Sh_0)^2\lambda_P X^2}{(2\lambda_P^2+1)^2\sqrt{(2\lambda_P^2+1)}}\right]I_{6-1},$$

$$I_{6-1} = \left[I_{6-1-1} - \alpha_0\lambda_P I_{6-1-2}\right.$$
$$\left.+ \frac{\alpha_0(2\lambda_P^2+1)}{\lambda_P}I_{6-1-3} + 2\alpha_0\lambda_P I_{6-1-4}\right],$$

$$I_{6-1-1} = J_{5-1-3},$$
$$I_{6-1-2} = J_{5-1-3},$$

$$I_{6-1-3} = \left[0.5 + \frac{0.234}{(2\lambda_P^2+1)^2} + \frac{0.154}{(2\lambda_P^2+1)^4} + \frac{0.038}{(2\lambda_P^2+1)^6} + \frac{0.008}{(2\lambda_P^2+1)^8}\right],$$

$$I_{6-1-4} = J_{5-1-3}.$$

$$C_{P6-2} = 0.$$

$$C_{P6-3} = 2\pi D_3\left[\frac{\overline{V_{yc}\nu_{nc}\dot{\omega}_z\sin\vartheta}}{U_0^3U_c}\right]$$

$$= 2\pi D_3\left[\frac{4\sqrt{2}(Sh_0)^4\alpha_0\lambda_P^4X^2}{(2\lambda_P^2+1)^3\sqrt{(2\lambda_P^2+1)}}\right]I_{6-3},$$

$$I_{6-3} = \left[-\frac{(2\lambda_P^2+1)^2}{8(Sh_0)^2\lambda_P^4X^2}I_{6-3-1} - \frac{(2\lambda_P^2+1)}{4(Sh_0)^2\lambda_P^4X^2}I_{6-3-2} + \frac{\alpha_0(2\lambda_P^2+1)^3}{16(Sh_0)^2\lambda_P^5X^2}I_{6-3-3} + \frac{\alpha_0(2\lambda_P^2+1)^2}{8(Sh_0)^2\lambda_P^3X^2}I_{6-3-4} + \frac{\alpha_0(2\lambda_P^2+1)}{4(Sh_0)^2\lambda_P^3X^2}I_{6-3-5} - \frac{\alpha_0^2(2\lambda_P^2+1)^3}{16(Sh_0)^2\lambda_P^4X^2}I_{6-3-6} - \frac{\lambda_P}{\alpha_0(2\lambda_P^2+1)}I_{6-3-7} + I_{6-3-8} - \frac{\alpha_0(2\lambda_P^2+1)}{4\lambda_P}I_{6-3-9} + \frac{\alpha_0^2(2\lambda_P^2+1)^2}{8\lambda_P}I_{6-3-10} + \frac{\lambda_P^2}{(2\lambda_P^2+1)}I_{6-3-11} - \lambda_P I_{6-3-12} + \frac{\alpha_0^2(2\lambda_P^2+1)}{4}I_{6-3-13} - \frac{\alpha_0\lambda_P}{2}I_{6-3-14} + \frac{\alpha_0^2(2\lambda_P^2+1)}{2}I_{6-3-15} - \frac{\alpha_0^3(2\lambda_P^2+1)^2}{8\lambda_P}I_{6-3-16}\right],$$

$$I_{6-3-1} = J_{5-3-4},$$
$$I_{6-3-2} = J_{5-3-5},$$
$$I_{6-3-3} = J_{5-3-6},$$
$$I_{6-3-4} = J_{5-3-4},$$
$$I_{6-3-5} = J_{5-3-5},$$
$$I_{6-3-6} = J_{5-3-6},$$

$$I_{6-3-7} = \left[0.5 + \frac{1.5463}{(2\lambda_P^2+1)^2} + \frac{3.1421}{(2\lambda_P^2+1)^4} + \frac{4.6172}{(2\lambda_P^2+1)^6} + \frac{5.1789}{(2\lambda_P^2+1)^8} + \frac{4.0626}{(2\lambda_P^2+1)^{10}}\right],$$

$$I_{6-3-8} = \left[0.5 + \frac{0.9844}{(2\lambda_P^2+1)^2} + \frac{1.4663}{(2\lambda_P^2+1)^4} + \frac{1.561}{(2\lambda_P^2+1)^6} + \frac{1.1845}{(2\lambda_P^2+1)^8}\right].$$

$$I_{6-3-9} = J_{5-1-3},$$
$$I_{6-3-10} = J_{5-1-7},$$
$$I_{6-3-11} = J_{5-8-2},$$
$$I_{6-3-12} = I_{6-3-8},$$
$$I_{6-3-13} = J_{5-1-3},$$
$$I_{6-3-14} = I_{6-3-8},$$
$$I_{6-3-15} = I_{6-3-9},$$
$$I_{6-3-16} = J_{5-1-7}.$$

$$C_{P6-4} = 2\pi D_3\left[\frac{\overline{V_{yc}\nu_{nc}\dot{\nu}_{nc}\sin\vartheta}}{U_0^3U_c}\right]$$

$$= 2\pi D_4\left[\frac{4\sqrt{2}(Sh_0)^4\alpha_0\lambda_P^6X^3}{(2\lambda_P^2+1)^4\sqrt{(2\lambda_P^2+1)}}\right]I_{6-4},$$

$$I_{6-4} = \left[\frac{(2\lambda_P^2+1)^3}{8(Sh_0)^2\lambda_P^6X^2}I_{6-4-1} + \frac{(2\lambda_P^2+1)^2}{8(Sh_0)^2\lambda_P^6X^2}I_{6-4-2} - \frac{1}{\alpha_0\lambda_P}I_{6-4-3} + I_{6-4-4} + \frac{2}{(2\lambda_P^2+1)}I_{6-4-5} + \frac{(2\lambda_P^2+1)}{2\lambda_P^2}I_{6-4-6} + \frac{(2\lambda_P^2+1)}{\lambda_P^2}I_{6-4-7} + \frac{2}{\lambda_P^2}I_{6-4-8} - \frac{\alpha_0(2\lambda_P^2+1)^4}{16(Sh_0)^2\lambda_P^7X^2}I_{6-4-9} - \frac{(2\lambda_P^2+1)^3}{8(Sh_0)^2\lambda_P^6X^2}I_{6-4-10} + \frac{\alpha_0(2\lambda_P^2+1)^4}{16(Sh_0)^2\lambda_P^7X^2}I_{6-4-11} + \frac{\alpha_0(2\lambda_P^2+1)^3}{8(Sh_0)^2\lambda_P^5X^2}I_{6-4-12} + \frac{\alpha_0(2\lambda_P^2+1)^2}{4(Sh_0)^2\lambda_P^5X^2}I_{6-4-13} - \frac{\alpha_0(2\lambda_P^2+1)^2}{2\lambda_P^3}I_{6-4-14} - \frac{\alpha_0(2\lambda_P^2+1)^2}{4\lambda_P^3}I_{6-4-15} - \frac{\alpha_0(2\lambda_P^2+1)}{2\lambda_P}I_{6-4-16} - \frac{\alpha_0(2\lambda_P^2+1)}{\lambda_P}I_{6-4-17} - \frac{\alpha_0^2(2\lambda_P^2+1)^3}{8(Sh_0)^2\lambda_P^4X^2}I_{6-4-18} - \frac{\alpha_0^2(2\lambda_P^2+1)^4}{16(Sh_0)^2\lambda_P^6X^2}I_{6-4-19} + \frac{\alpha_0^2(2\lambda_P^2+1)^3}{8\lambda_P^4}I_{6-4-20}\right],$$

$$I_{6-4-1} = I_{6-1-3},$$
$$I_{6-4-2} = J_{5-3-5},$$
$$I_{6-4-3} = J_{5-8-2},$$
$$I_{6-4-4} = J_{5-8-2},$$

$$I_{6-4-5} = J_{5-8-3},$$
$$I_{6-4-6} = J_{5-8-5},$$
$$I_{6-4-7} = J_{5-8-5},$$
$$I_{6-4-8} = \left[0.5 + \frac{0.5625}{(2\lambda_P^2+1)} + \frac{1.5463}{(2\lambda_P^2+1)^2} + \frac{1.6756}{(2\lambda_P^2+1)^3} + \frac{3.1421}{(2\lambda_P^2+1)^4} + \frac{3.1924}{(2\lambda_P^2+1)^5} + \frac{4.6172}{(2\lambda_P^2+1)^6} + \frac{8.482}{(2\lambda_P^2+1)^7} + \frac{5.1789}{(2\lambda_P^2+1)^8} + \frac{4.1323}{(2\lambda_P^2+1)^9} + \frac{4.0686}{(2\lambda_P^2+1)^{10}}\right],$$
$$I_{6-4-9} = J_{5-1-8},$$
$$I_{6-4-10} = J_{5-3-4},$$
$$I_{6-4-11} = J_{5-3-6},$$
$$I_{6-4-12} = J_{5-3-4},$$
$$I_{6-4-13} = J_{5-3-5},$$
$$I_{6-4-14} = J_{5-1-3},$$
$$I_{6-4-15} = J_{5-1-3},$$
$$I_{6-4-16} = J_{5-8-5},$$
$$I_{6-4-17} = J_{5-8-5},$$
$$I_{6-4-18} = J_{5-1-7},$$
$$I_{6-4-19} = J_{5-3-8},$$
$$I_{6-4-20} = J_{5-1-7}.$$
$$C_{P6-5} = 2\pi D_5\left[\frac{V_{yc}\dot{\nu}_{nc}\omega_z\sin\vartheta}{U_0^3 U_c}\right] = 2\pi D_5\left[\frac{\sqrt{2}(Sh_0)^2\alpha_0}{2(2\lambda_P^2+1)\sqrt{(2\lambda_P^2+1)}}\right]I_{6-5},$$
$$I_{6-5} = \left[I_{6-5-1} - \frac{8(Sh_0)^2\lambda_P^5X^2}{\alpha_0(2\lambda_P^2+1)^3}I_{6-5-2} + \frac{4(Sh_0)^2\lambda_P^4X^2}{(2\lambda_P^2+1)^2}I_{6-5-3} + \frac{8(Sh_0)^2\lambda_P^4X^2}{(2\lambda_P^2+1)^2}I_{6-5-4} + \frac{32(Sh_0)^2\lambda_P^6X^2}{(2\lambda_P^2+1)^3}I_{6-5-5} + \frac{4(Sh_0)^2\alpha_0\lambda_P^3X^2}{(2\lambda_P^2+1)}I_{6-5-6} + \frac{2(Sh_0)^2\alpha_0\lambda_P^3X^2}{(2\lambda_P^2+1)}I_{6-5-7} - \alpha_0\lambda_P I_{6-5-8} + \frac{8(Sh_0)^2\lambda_P^6X^2}{(2\lambda_P^2+1)^3}I_{6-5-9} - \frac{8(Sh_0)^2\alpha_0\lambda_P^5X^2}{(2\lambda_P^2+1)^2}I_{6-5-10} - \frac{8(Sh_0)^2\alpha_0^2\lambda_P^4X^2}{(2\lambda_P^2+1)}I_{6-5-11}\right],$$
$$I_{6-5-1} = J_{5-1-7},$$
$$I_{6-5-2} = J_{5-8-2},$$
$$I_{6-5-3} = I_{6-3-8},$$
$$I_{6-5-4} = I_{6-3-8},$$
$$I_{6-5-5} = I_{6-3-7},$$
$$I_{6-5-6} = J_{5-1-3},$$
$$I_{6-5-7} = J_{5-1-3},$$
$$I_{6-5-8} = J_{5-1-7},$$
$$I_{6-5-9} = I_{6-3-7},$$
$$I_{6-5-10} = I_{6-3-8},$$
$$I_{6-5-11} = J_{5-1-3}.$$
$$C_{P6-6} = 0,$$
$$C_{P6-7} = 0.$$
$$C_{P6-8} = 2\pi D_8\left[\frac{V_{yc}\dot{\nu}_{nc}^2\sin\vartheta}{U_0^3U_c^2}\right] = 2\pi D_8\left[\frac{2\sqrt{2}(Sh_0)^4\lambda_P^3X^2}{(2\lambda_P^2+1)^3\sqrt{(2\lambda_P^2+1)}}\right]I_{6-8},$$
$$I_{6-8} = \left[\frac{\alpha_0^2(2\lambda_P^2+1)^3}{8(Sh_0)^2\lambda_P^4X^2}I_{6-8-1} + I_{6-8-2} + \frac{4}{(2\lambda_P^2+1)}I_{6-8-3} - \frac{\alpha_0(2\lambda_P^2+1)}{\lambda_P}I_{6-8-4} + \frac{\alpha_0^2(2\lambda_P^2+1)^2}{4\lambda_P^2}I_{6-8-5} - 2\alpha_0\lambda_P I_{6-8-6} + \frac{\alpha_0^2(2\lambda_P^2+1)}{2}I_{6-8-7} + \alpha_0^2(2\lambda_P^2+1)I_{6-8-8} - \frac{\alpha_0^3(2\lambda_P^2+1)^2}{2\lambda_P} - \frac{\alpha_0^3(2\lambda_P^2+1)^3}{2(Sh_0)^2\lambda_P^3X^2}I_{6-8-10} + \alpha_0\lambda_P I_{6-8-11} - \frac{4\alpha_0\lambda_P}{(2\lambda_P^2+1)}I_{6-8-12} + 2\alpha_0^2\lambda_P^2 I_{6-8-13}\right],$$
$$I_{6-8-1} = J_{5-1-8},$$
$$I_{6-8-2} = J_{5-8-6},$$
$$I_{6-8-3} = J_{5-8-7},$$
$$I_{6-8-4} = J_{5-8-11},$$
$$I_{6-8-5} = J_{5-3-4},$$
$$I_{6-8-6} = I_{6-3-8},$$
$$I_{6-8-7} = J_{5-1-3},$$
$$I_{6-8-8} = I_{6-3-11},$$
$$I_{6-8-9} = J_{5-1-7},$$
$$I_{6-8-10} = J_{5-1-8},$$
$$I_{6-8-11} = I_{6-3-8},$$
$$I_{6-8-12} = J_{5-8-7},$$
$$I_{6-8-13} = I_{6-3-8}.$$

For purely linear oscillations ($\vartheta = 0$),

$$C_{T5-1} = -2\pi D_1\left(\frac{1}{2\lambda_P^2}\right),$$
$$C_{T5-2} = C_{T5-3} = C_{T5-4} = C_{T5-5} = C_{T5-6} = C_{T5-7} = C_{T5-9} = C_{T5-10} = 0,$$

$$C_{T5-8} = -2\pi D_8 \left\{ \frac{1}{(2\lambda_P^2+1)} \left[1 + \frac{0.5}{(2\lambda_P^2+1)} + \frac{0.5}{(2\lambda_P^2+1)^2} + \frac{0.375}{(2\lambda_P^2+1)^3} + \frac{0.17}{(2\lambda_P^2+1)^4} + \frac{0.073}{(2\lambda_P^2+1)^5} \right] \right\},$$

$$C_{P6} = 0.$$

In the above formulas, X is the relative distance from the axis of rotation to the center of the wing ($X = x/b$), x is the absolute distance from the axis of rotation to the center of the wing (positive if the axis of rotation is located closer to the back edge and negative if the axis is closer to the frontal edge), $\lambda_P = \frac{U_0}{\omega y_0}$, y_0 is the amplitude of linear oscillations of the wing, $\omega = 2\pi f$, and f is frequency.

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